

# ECONOMICS OF VARIABILITY IN DAIRY RATIONS

Normand R. St-Pierre, Ph.D., P.A.S. and W. P. Weiss, Ph.D.  
The Ohio State University

## Summary

Feed variation can result in lost productivity and profitability. The control of diet variation becomes an application where statistical process control and statistical decision theory can be put to valuable use. Our objective in this paper is to review the sources of feed variation, their impact on animal productivity, and practical ways to reduce the variation in nutritional characteristics of diets.

There are two types of diet variations: 1) abrupt changes in composition as when receiving a new batch of feed, and 2) random variation because feed particles are not nutritionally uniform. The control of variation in diet composition must be initiated before diet formulation. This requires periodical chemical analyses of feedstuffs. What should be analysed, at what frequency, and when the diet should be modified has been studied as a renewal reward process. Results showed that the optimal sampling pattern varies across feeds, nutrients, and herd size. Important practices include:

- 1) Maintaining separate inventories of feeds with different nutritional characteristics,
- 2) Sourcing ingredients from a single source, and
- 3) Purchasing commercial feeds from a manufacturer with an effective quality control program.

Variation in diet composition can be greatly affected by formulation. With simple nutrients, i.e., those that can be expressed as a proportion of dry matter (**DM**) and that do not interact with other nutrients (e.g., crude protein; **CP**), the contribution of an ingredient to diet variance changes with the square of its inclusion rate. For complex or composite nutrients (e.g., rumen undegradable protein; **RUP**), diet variance is a complex function of multiple covariances. Approximation formulas exist, but are generally very inaccurate. Monte Carlo simulation methods have been used successfully in these instances. Unresolved issues exist related to the identification of response functions to nutrient variation as well as obtaining reasonable estimates of variances and covariances for each feedstuff. In general, increasing the number of ingredients in the diet, and increasing the use of ingredients with low variability lead to less variable diets. Commercial feeds should be less variable than commodities and; therefore, carry an additional economic value.

## Introduction

A fundamental rule regarding diet formulation is that one never knows the true value of anything. Although we have reasonably accurate estimates of the *average* requirements for most nutrients, we have less certainty regarding nutrient requirements of a specific herd or animal under specific circumstances. We have equations that accurately estimate the *average* dry matter intake (**DMI**) for groups of cows, but estimating intake accurately of a specific cow is more difficult. We have developed several good analytical procedures to measure the concentrations of many nutrients in feeds and tables are available that contain the *average* nutrient composition of all feeds commonly fed to dairy cows. However, biological and manufacturing variation, variation caused by sampling, and variation in analytical measurements can be substantial so that concentrations of nutrients within a specific feedstuff may be quite different from the average. Does all this uncertainty mean that we should give up on ration formulation and feed analysis? The answer to that question is obviously, no. However, the

uncertainty associated with feed analysis and ration formulation must be understood and addressed. With proper sampling techniques, adequate number of samples, and appropriate data handling, one can reduce the uncertainty associated with feed analysis data. The objective of this paper is to discuss expected variation in feed composition, factors affecting variation, and methods one can use to increase the reliability of feed analysis data. The following discussion is appropriate for all feeds, but the paper will concentrate on grains and by-products.

## Elementary Statistics

We need to start thinking about feed composition data in terms of probabilities rather than actual, absolute concentrations. In other words, how confident should you be that the analytical value received from a laboratory actually represents the true concentration of a nutrient in a feed? Because we are working with probabilities, a basic understanding of some statistical principles and terminology is needed.

## Populations and Samples

The ultimate goal of feed analysis is to obtain an analytical value from a sample that reflects the actual value of a 'population'. Populations can be quite different depending on the application. For example, a population can be a truckload of distillers dried grains, or all the distillers dried grains produced by a specific distillery, or perhaps all the distillers dried grains produced in the country. In statistical terms, a population is loosely defined as a large set from which samples can be taken. If distillers grains from a single distillery were sampled extensively, we would have a good estimate of the average nutrient composition of distillers grains produced at that plant. However, since other distilleries were not sampled we should be very hesitant to extrapolate the data obtained from a single distillery (i.e., a narrow population) to the larger population of all distilleries.

## Central Tendency and Dispersion

A population can be represented by a set of observations or samples. Because of inherent variation among the particles making a feed and because of variation caused by sampling and analytical procedures, we know that all the sample values will not be the same. Rather than one single value, one can obtain a distribution of values. The two most important pieces of information we need to obtain from a set of samples are a measure of central tendency and a measure of dispersion. For observations that follow a normal statistical distribution, the mean (in this discussion average and mean will be used interchangeably) is the best measure of central tendency. The mean of a normal distribution is not the absolute 'right' answer, but rather it is the value that has the lowest probability of being substantially wrong (i.e., it is the most likely value – the expected value). The concentrations of most nutrients in plant-based feedstuffs fit approximately a normal distribution; therefore the mean is the best measure of central tendency for those nutrients. With a normal distribution, approximately one-half of the samples have values lower than the mean and one-half have concentrations higher than the mean. The concentrations of trace minerals and a few other chemicals, such as ether extracts - or fats - in plant-based feeds, often have a skewed distribution (a few observations will have very high concentrations). With this type of distribution, the mean still represents the *expectation*, but it overestimates the central tendency. The median (the value at which half the observations are higher and half are lower) is the best measure of central tendency for this type of distribution.

Although many people are familiar with and often use measures of central tendency (i.e., the mean) in ration formulation, fewer people consider or use measures of dispersion in ration formulation. In simple terms, a measure of dispersion should be used to determine how much confidence one has when using a mean value. When a distribution of values has a large dispersion, the probability of being substantially wrong when using the mean increases. For a normal distribution the most common measure of dispersion is the standard deviation (**SD**). In a normal distribution, approximately 38 % of all observations are within  $\pm 0.5$  SD units of the mean, 68 % of all observations are within  $\pm 1$  SD of the mean, and approximately 95 % of the observations are within  $\pm 2$  SD of the mean. For example, if the mean concentration of CP in a population of brewers dried grains is 25 % and the SD is 2 we would expect that about 68 % of the samples from that population would contain between 23 and 27 % CP and 95 % of the samples would contain between 21 and 29 % CP. This also means that about 5 % of the samples would contain less than 21 or more than 29 % CP. The smaller the SD, relative to the mean, the less likely it is that using the mean value will cause a substantial error in diet formulation.

## Sources of Variation

Understanding potential sources of variation in feed composition data helps determine which data to use and how to use it. The nutrient composition of grains and by-products can be influenced by plant genetics (hybrid, variety, etc) and growing conditions (drought, climate, soil fertility, etc.). In addition, the composition of by-products is affected by manufacturing techniques. The above sources of variation are considered fixed, i.e., they can be described and replicated). In statistical quality control jargon, they are labelled as *assignable causes*. Hybrid X may have been genetically selected to produce corn grain with higher than average concentrations of protein. Distillery Y might dry their distillers grains at very high temperatures causing high concentrations of acid detergent insoluble protein (**ADIP**). A drought may reduce kernel size; thereby increasing the concentration of fiber in corn grain. Another possible fixed source of variation is the analytical lab. Although great progress has been made in standardizing methods, labs may use different analytical techniques to measure nutrients. If lab A measures neutral detergent fiber (**NDF**) using sulfite, but another lab does not, the NDF concentrations will differ between the labs because of procedure.

Other sources of variation are considered random. We do not know why the values differ,

they just do. If you sample a load of brewers grains 10 times and send those 10 samples to a lab, you will probably get back 10 slightly different concentrations of CP. The variation could be caused by variation within the load of brewers grain or it could be caused by random errors at the lab. The causes of the variation are unknown. They are referred to in quality control jargon as *unassignable causes*.

Ideally, random variation would be considered within population variation and fixed variation would be considered as variation between populations. For example, because of manufacturing differences, distillers grains from distillery X has consistently higher NDF concentrations than distillers grains from distillery Y. If distillers grains from X and Y were considered separate populations, the SD within each population would be lower than the SD when the results from both distilleries are combined. Because of blending grains and multiple sources of feedstock for manufacturing facilities, many fixed sources of variation become blurred (you will not know the variety of soybeans used to make the soybean meal you purchased or whether the gluten feed you purchased was made from drought-stressed corn grain). In these situations, the fixed sources of variation (assignable causes) become random sources (unassignable causes) resulting in an increase in the within population variation. Nonetheless, accounting for as many fixed sources of variation as possible by defining separate populations will reduce the dispersion of the data and reduce the potential of being substantially wrong when using the mean.

### **Expected Variation in Nutritional Composition Of Feeds**

The largest publicly-available data base of feed composition in the USA can be found in the NRC dairy publication (NRC, 2001). That database contains means, SD, and the number of samples for measured nutrients in most common feedstuffs used in North America. The data used to calculate those means and SD came from a wide array of sources. Samples came from across the US and over several years. For some feeds and nutrients, the number of samples used to calculate the mean and SD is quite limited and those values should be used with caution. For other feeds, the sample size is quite large and the mean and SD are probably good estimates for the broad population from which the samples were drawn. However, it is important to remember that the broad population represented in the NRC tables may not be a good estimate for a specific source of a feed. Kertz (1998) also provides data on variation in nutrient composition of a limited number of feeds.

Based on expected variation, feeds can be classified as having low, moderate, or high variability. Feeds with generally low variability include corn grain, sorghum grain, and perhaps barley (Table 1). Feeds with the largest variability in composition are by-products that are usually not a direct co-product of manufacturing. For example, potato waste has extremely high variability because it may include cull potatoes, potato peels, waste products from the manufacturing of potato products for human consumption, rejected product, etc. Millrun, corn screenings, and cannery waste are other examples of feeds that are not well-defined and would be expected to have high variability; even when they come from the same originating source. Feeds with moderate variability include most feeds that would be considered co-products rather than by-products. Distillers grains, brewers grains, and corn gluten feed are end products of alcohol, beer, and corn sweetener production. Because production of these products is generally well-controlled, the composition of the resulting co-product can be relatively constant within a production facility. The forages in Table 1 have moderate variability, but note how variation decreases when a more exact definition of the forages is used (alfalfa silage vs. mid-maturity alfalfa silage).

Net energy for lactation ( $NE_L$ ) and metabolizable protein ( $MP$ ) are arguably the most important nutrients used in dairy diet formulation, but they present unique problems in terms of variation. Those nutrients are not measured by laboratories but are calculated from numerous variables, some of which are measured while others are estimated. The complexity and nonlinearity of the new models used for diet evaluation and/or balancing (NRC, 2001) make it impossible to calculate directly (i.e., using an equation) the variation in  $NE_L$  and  $MP$  of the diet that is attributable to the variation in the nutritional composition of the feeds making up the diet. In fact, there is enough nutrient interaction in the equations used that it is also impossible to calculate the variation in  $NE_L$  and  $MP$  content of a specific feed given its own nutritional variation in analytical nutrients. However, we can simulate this variation and examine its effect through multiple replications, using modern high speed computers and a method called Monte Carlo simulation. A new software program, called *Ping Pong*<sup>TM</sup> has been developed at Ohio State to study the effects of nutrient variation in feedstuffs on the variance in  $NE_L$  and  $MP$  of the diet (Beta version available for free at [www.sesamesoft.com](http://www.sesamesoft.com)). A similar application called *Skip-e*<sup>TM</sup> is available from H. J. Baker & Sons. An example of variation in  $NE_L$  of alfalfa hay is shown in Figure 1.

**Table 1.** Average (Avg) concentrations and standard deviations (SD) for CP, NDF, and ether extract (EE) in selected feeds. Data are from NRC (2001) and represent very diverse populations.

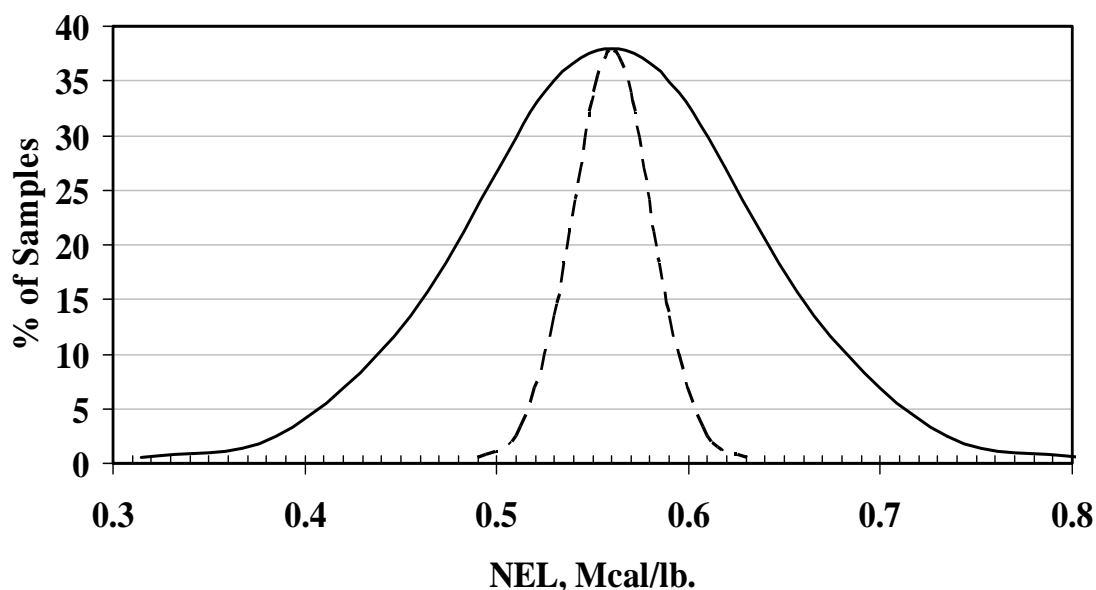
	CP		NDF		EE	
	Avg	SD	Avg	SD	Avg	SD
<b>Grains</b>						
Barley	12.4	2.1	20.8	8.6	2.2	0.6
Corn	9.4	1.3	9.5	2.3	4.2	1.0
Sorghum	11.6	1.8	10.9	5.0	3.1	0.8
<b>By-products</b>						
Wet brewers	28.4	4.0	47.1	6.8	5.2	1.6
Corn gluten feed	23.8	5.7	35.5	6.8	3.5	1.1
Dry distillers grain	29.7	3.3	38.8	7.8	10.0	3.4
Potato waste	10.5	8.4	22.1	14.3	10.8	7.8
Rice bran	15.5	2.2	26.1	6.8	15.2	4.2
Soyhulls	13.9	4.6	60.3	7.4	2.7	1.4
Soybean meal-48	53.8	2.1	9.8	5.6	1.1	0.4
Wheat midds	18.5	2.1	36.7	7.5	4.5	1.3
<b>Forages</b>						
Corn silage	8.8	1.2	45.0	5.3	3.2	0.5
Alfalfa silage (AS)	20.6	3.0	45.7	6.5	3.1	0.7
Mid-maturity AS	21.9	1.8	43.2	1.5	2.2	0.3

If NRC data (Table 2) are used (a broad population), the average  $NE_L$  is 1.23 Mcal/kg with a SD of 0.15. If samples are from a well-defined population (e.g., hay from a single farm and cutting, Table 2), the average  $NE_L$  is still 1.23 Mcal/kg but the SD is now 0.04.

To increase the accuracy of ration formulation, feeds with moderate and high variability in composition must be sampled and analysed routinely and the data generated must be used correctly. An accurate estimate of SD for a specific feedstuff can be extremely useful in ration formulation. The SD should be considered when deciding on ration *safety* factors. The SD in the NRC table is a function of inherent variation in composition of the grain or feedstock, lab-to-lab variation, variation among manufacturing processes, and many other sources of variation. If no other measure of dispersion is available, the SD in the NRC table can be used; however, one must remember that for many feeds, the actual variation

could be substantially less than the SD in the NRC table (Table 3).

Several common feeds were sampled and analysed over a one year period in California (DePeters et al., 2000). All analysis were conducted at a single lab and for the feeds that will be discussed, all samples within a feed came from the same production facility. A similar type study was conducted in Missouri (Belyea et al., 1989). Dried distillers grains were sampled in both studies. The calculated distributions of CP concentrations are shown in Figure 2 for the two studies and for NRC data. Mean concentration of CP was very similar for the three data sets (29.7, 30.6, and 31.2 % of DM for NRC, MO, and CA; respectively). However, dispersion differed greatly. The SD for NRC, MO, and CA were 3.3, 1.6, and 0.6; respectively. Based on the means and considering typical dietary inclusion rates for distillers grains, essentially the same concentration of dietary CP



**Figure 1.** Expected variation in  $NE_L$  of alfalfa hay as calculated by the software Ping Pong™. The solid line represents a broad population (NRC, 2001). The dashed line represents a very well defined population (see Table 2 for standard deviations used in the simulation).

would be obtained regardless of the source of the data. However, because the SD is substantially lower when all samples were obtained from a single source, one would be much less likely to make a substantial error in formulation (i.e., diet is actually deficient in CP) when the mean value is used if the sample is from a limited, rather than a broad population. Not all feeds or nutrients follow the pattern shown for distillers grains in Figure 2. In Figure 3, for example, we show the distribution of CP concentrations for rice bran from the NRC data set (broad population) and from Belyea et al.

(1989; limited population). If one used the mean concentration of CP from NRC for rice bran obtained from the particular production facility sampled in the Missouri study, the CP concentration would be substantially underestimated, resulting in increased protein supplementation costs. For nutrients that are routinely measured, means obtained from a broad population (e.g., NRC) should be used only when other data specific to a limited population are not available.

**Table 2.** Standard deviations of nutrients for two alfalfa hays used in the Monte Carlo simulation. The commodity alfalfa data are from NRC, the lab-tested alfalfa represents a well-defined population.

Nutrient	Commodity Alfalfa	Lab-tested Alfalfa
DM, %	1.4	0.5
CP, %	2.6	0.3
NDF, %	6.3	0.8
Ether extract, %	0.5	0.5
Ash, %	1.2	0.2
Lignin, %	0.9	0.2
ADICP, %	0.4	0.4
NDICP, %	0.9	0.9

**Table 3.** Average (Avg) concentrations and standard deviations for selected nutrients and selected feeds. The California data are from DePeters et al. (2000) and the Missouri data are from Belyea et al. (1989). Within experiment and feed, samples originated from the same production facility (i.e., limited populations). These values should be compared to those in Table 1 (a broad population).

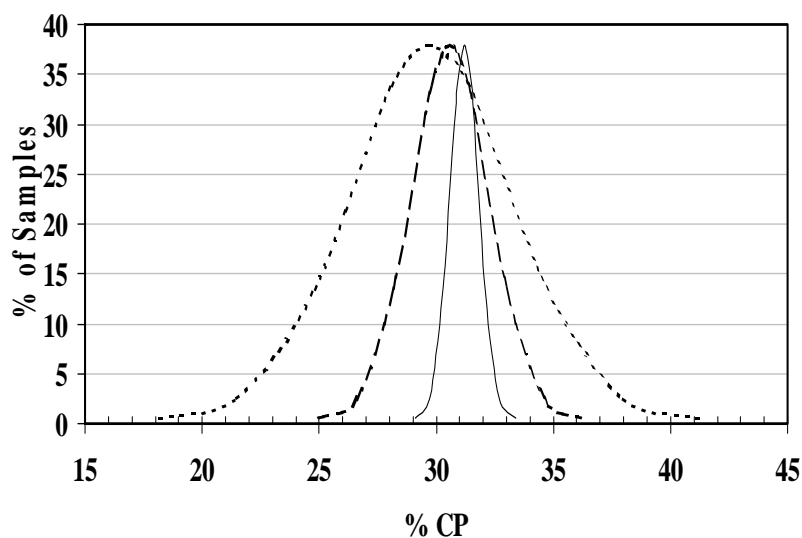
Source Selected Feed	CP		NDF		EE	
	Avg	SD	Avg	SD	Avg	SD
<b>CA</b>						
Brewers grain, wet	27	2.2	37.3	3.4	6.3	0.4
Corn gluten feed, wet	22.9	4.3	38.8	3.8	3.4	0.4
Distillers grain, dried	31.2	0.6	35.6	8.2	13	1.3
<b>MO</b>						
Corn gluten feed, dry	23.3	1.4	51.9	2.3	6.6	1.9
Distillers grain, dried	30.6	1.4	33	1.5	7.4	0.9
Rice bran	19.1	0.4	21.8	1.3	17.3	1.9
Soybean hulls	11.8	0.2	72.5	0.8	0.8	0.3

### Handling Variation in Feed Composition

Variation in feed composition is handled differently depending upon whether a given feed is best conceptualized as the outcome of a batch process vs. a continuous process.

### Batch-Process Feedstuffs

Feeds in this category are handled in lots such as trucks and train cars. The manufacturing may be a continuous process, but their use is generally best described as a batch process. Most feed commodities used by commercial feed manufacturers fall into this category. They are



**Figure 2.** Distributions of crude protein concentrations in dried distillers grains. The small dashed line represents data from a nationwide population (NRC, 2001). The large dashed line represents samples from a single source in Missouri (Belyea et al., 1989) and the solid line represents samples from a single source in California (DePeters et al., 2000). Although mean concentrations were similar among populations, note that dispersion is substantially less for the limited populations (CA and MO) compared to the broad population (NRC). Distributions were calculated based on the mean and SD.

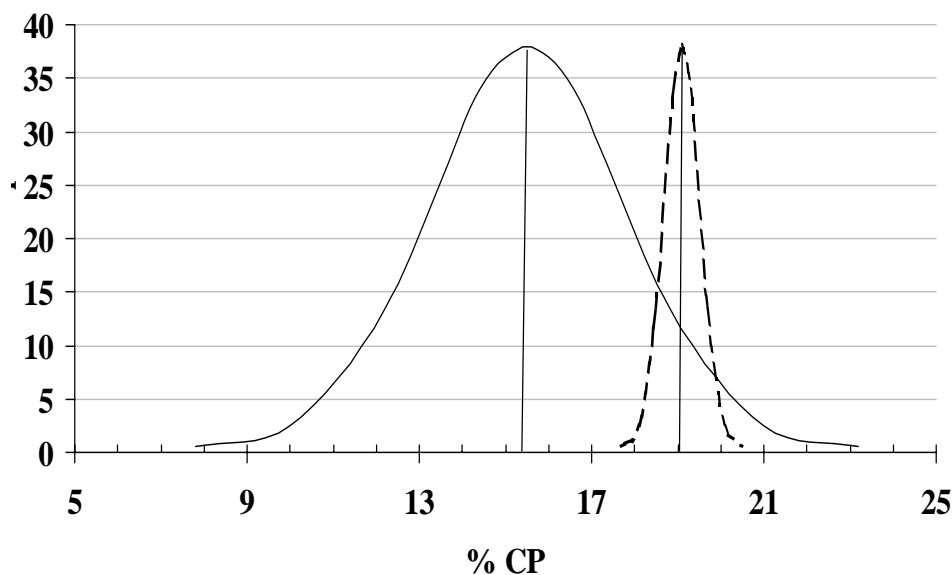
characterized by small variation within lots, and small to large variation between lots.

Feeds with low expected variability between lots do not have to be analysed routinely and, in some cases, not at all. Sampling and analytical errors become relatively small when large numbers of samples are analysed. For these feeds, a mean derived from a large number of samples may actually be better than a single observation or a mean from a small set of samples. For these feeds, book values can be used unless one has good reason to believe that a particular feed is different (for example, if you grow or buy high oil corn, the mean values for regular corn would not be appropriate).

For feeds with moderate or high variability in nutrient composition, routine feed sampling and analysis is essential. Although most people realize this, it is often not done because by the time they get the report back from the lab, the load has been fed. If this is your opinion, you are not using the analytical data correctly. As stated above, we need to think in terms of probabilities, not absolute numbers. You should be sampling and analysing load samples to obtain estimates of mean composition and SD; the values obtained from a single load sample are not that important. The frequency of sampling depends on the expected variation and how much error one is willing to accept. Populations with large variation require more sampling to obtain accurate estimates. I cannot give you a specific number of samples

needed because it varies depending on the nutrient of interest (e.g., the number of samples needed to obtain accurate estimates of the mean and SD for CP is usually less than that needed for NDF) and the population. As a general guideline 10 or more samples of a given population is reasonable. For highly variable feeds more samples are desirable.

The approach followed by many nutritionists is to sample a load of feed, have it analysed, and then formulate a diet based on that information. When a new analysis is obtained, the previous data are eliminated and a new diet is formulated based on the new composition. The inherent assumption underlying this practice is that the new data better represents the feed than did the old data. This may or may not be true. When new analytical data are obtained, the user should ask one simple question: is there an identifiable reason why the composition changed? Possible answers to that question include: the supplier changed, the distillery changed production methods, or probably most commonly, I don't know. If you cannot think of a good reason for the composition change, the change may simply be a random event. The difference could be caused by load-to-load random variation, by within load (i.e., sampling) variation, or both. In this case, the new number may be no better than the old number, but the mean of the two numbers has the lowest probability of being substantially wrong. The mean, rather than the new or old number should be used for ration formulation. Users should collate feed composition data using a spreadsheet or some other method and recalculate the mean and SD as



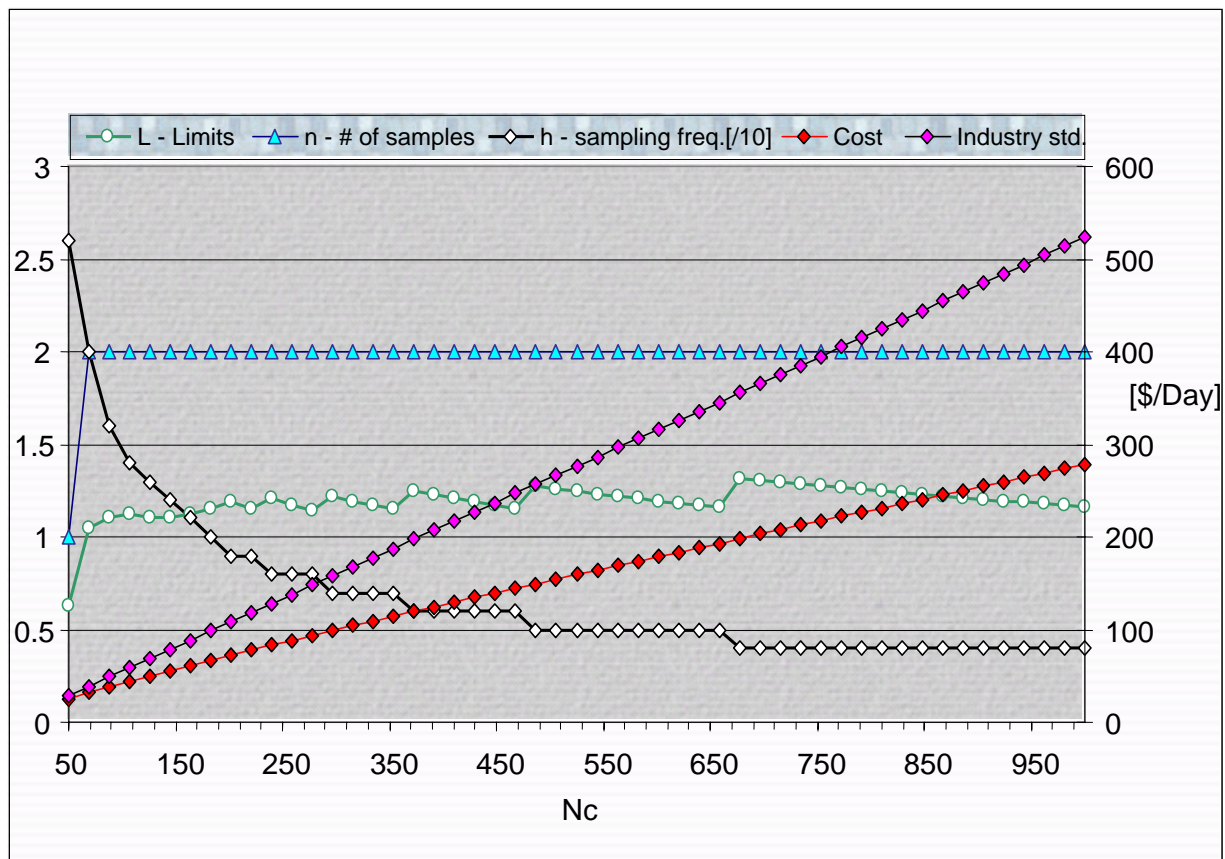
**Figure 3.** Distributions of crude protein concentrations in rice bran. The solid line represents data from a nationwide population (NRC, 2001) and the dashed line represents samples from a single source (Belyea et al., 1989). The means of the two populations are substantially different and the dispersion is much greater for the broad population. Distributions were calculated based on the mean and SD.

new data are collected. If you can come up with a logical reason why composition changed (i.e., a new population), then the new number should replace the old number and you start the process of collating data again. Statistical process control charts, such as the X-bar chart, can be used to identify composition changes resulting from assignable cause.

### Continuous Process Feedstuffs

Silages are excellent examples of feeds of this type. Silos are filled and, more importantly, unloaded in a somewhat continuous fashion. The composition of the silage remains relatively constant until the occurrence of an assignable cause: the hybrid or the variety changed, or the field from which the silage originated changed, etc. In this case, sampling for analysis is not done as much to determine means and SD but to identify

the occurrence of a shift in composition. Statistical process control tools such as X-bar and CUSUM charts are invaluable in this instance. The optimal sampling design, i.e., the number of samples to be taken ( $n$ ), the frequency of sampling ( $h$ ) and the departure from the mean on an X-bar chart ( $L$ ) expressed in SD units must be determined. In the USA, it has been customary to take one sample ( $n = 1$ ), once a month ( $h = 30$  d), and to automatically reformulate diets with the new data ( $L = 0$ ) or if the composition has changed by more than 2 SD ( $L = 2$ ). We have successfully modelled the process as a renewal reward process with 13 inputs that must be accounted for in the calculation of the total quality cost (St-Pierre and Cobanov, 2007). Figure 4 shows the optimal sampling design using current USA prices for herds of 50 to 1000 cows. From this figure, it is evident that the traditional sampling design is close to optimal in small herds of ~ 50 cows, but is grossly erroneous in large herds.



**Figure 4.** Effect of number of cows in the herd ( $N_c$ ) on optimal sampling design and total quality costs per day;  $L$  = the optimal control limits for an X-bar chart,  $n$  = the number of samples to be taken at each sampling time, and  $h$  = the sampling frequency expressed in days/10. The cost is the total quality cost per day. The industry standard curve is the total quality cost for the herd if a conventional sampling design is used (i.e.,  $n = 1$ ,  $h = 30$  d, and  $L = 2$ ). Note that for a herd of 1000 cows (becoming increasingly more frequent in the USA) the economic return to using an optimal sampling design (bottom diamond line) vs. the conventional design (upper diamond line) exceeds 250 US\$/d.



## Accounting for Feed Variation during Diet Formulation

As previously mentioned, the SD is an important statistic. It is an indicator of how wrong you could be. In Table 1, corn gluten feed has a mean CP concentration of 23.8 and a SD of 5.7. Assuming a normal distribution and totally random loads of corn gluten (i.e., not from a single source), approximately 16 % of the loads would have a CP concentration less than 18.1 % and 16 % of the loads would have a CP concentration greater than 29.5 %. If a particular load of corn gluten had 18 % CP and you used the mean concentration and corn gluten made up 10 % of the diet DM, the actual CP concentration of the diet would be about 0.6 % units lower than the formulated value. An error of this magnitude or larger would be expected 16 out of every 100 loads. If you are willing to accept this risk, then using the mean is the best option. However, if based on your experience, you conclude that milk production will drop 1 kg/cow/d (or some other number) if the diet contains 0.6 percentage units less CP than formulated and you are unwilling to accept that risk (even though this will happen only 16 % of the time), you need to adjust for variation. You can reduce your risk of substantially under feeding CP by *adjusting* the mean value based on its SD. Based on a normal distribution, if you use the mean minus 0.5 X SD, rather than the mean, you reduce the risk of making the error discussed above from 16 % of the time to 7% of the time. If you use the mean minus 1 SD unit, you reduce the risk of making the above error to just 2 % of the time. In the example above, mean CP for corn gluten was 23.8 (SD = 5.7). If I was willing to risk being substantially wrong 7 out of every 100 loads of corn gluten feed, I would use 23.8 - (0.5 x 5.7) or 21.0 % CP for corn gluten feed when I balanced the diet. If I only wanted to be substantially wrong 2% of the time, I would use 23.8 - 5.7 = 18.1 % CP. By using a lower CP concentration for corn gluten

feed, I have substantially decreased the probability of being substantially deficient in CP; however, I will be over supplementing CP most of the time. You will need to determine how much risk you are willing to accept and balance that against increased feed costs.

The problem with this approach is that it only considers variation in a single ingredient, but the nutrient composition of all ingredients in a diet will vary. What really matters is not the variation in a single ingredient, but rather the variation and mean for a diet. Methods of diet optimization when considering multiple nutrients from multiple variable sources are well defined and have been labelled as *chance-constrained programming* (St-Pierre and Harvey, 1986). These can be optimized using nonlinear programming techniques because the chance-constrained problem can be formulated as a deterministic nonlinear model. Unfortunately, this means that the convenient and widely available linear programming algorithms based on the simplex can no longer be used.

For measured nutrients such as CP, nutrients across feeds are independent (i.e., the level of CP in corn is independent of the level of CP in soybean meal, for example). In these instances, the variation in the total diet is a function of the *square* of the inclusion rate of each ingredient – which is where the nonlinearity enters the formulation model. That is, the contribution of a given feed to the total variance of the diet is quadrupled if its inclusion rate is doubled. Two kilograms of alfalfa has four times (2<sup>2</sup>) the CP variance of one kilogram (1<sup>2</sup>). In tables 4, 5 and 6 we present the expected CP and variance of the CP for a total mixed ration (TMR) under 3 different scenarios. The significant variance reduction from Table 5 to Table 6 is one major contribution of the commercial feed industry and has been calculated to be worth an additional 18 US\$/ton over and above the mean value of the nutrients based on USA prices.

**Table 4.** Expected mean crude protein level and variance of a simple TMR without feed analyses.

Feed	lbs/d (DM basis)	lbs CP	Variance (x 10,000)
Alfalfa silage	16.8	3.36	2964 (82%)
Corn silage	11.2	1.00	226
Ground shelled corn	12.9	1.26	67
Distillers dried grains	6.8	2.06	324
Soybean meal	3.6	1.95	25
Minerals-Vitamins	0.9	0	0
Total	52.2	9.63	3606

Standard Deviation = 0.60

For calculated nutrients such as  $NE_L$  and MP, the calculation of the variation of the diet becomes analytically intractable because nutrients are no longer independent across feedstuffs (i.e., the MP of soybean meal is dependent on the nutritional composition of corn). Monte Carlo techniques have been used to estimate the variation of these nutrients. The software programs mentioned previously (*Ping Pong*<sup>TM</sup> and *Skip-e*<sup>TM</sup>) can calculate variation in nutrient composition of diets if the user has information on variation in the individual ingredients. In addition, the programs

will calculate the implications of variation in nutrient composition on milk production. Currently, the software simulates the nutrient variation of a given diet, but it cannot optimize the diet. The computational problems associated with profit optimization in these circumstances are immense, partly because the objective function does not have a closed form. Thus derivative-based or gradient-based optimization methods cannot be used. We have tried non-parametric approaches such as the genetic algorithm with some success, but much work remains to be done.

**Table 5.** Expected mean crude protein level and variance of a simple TMR with forage samples analysed by a laboratory.

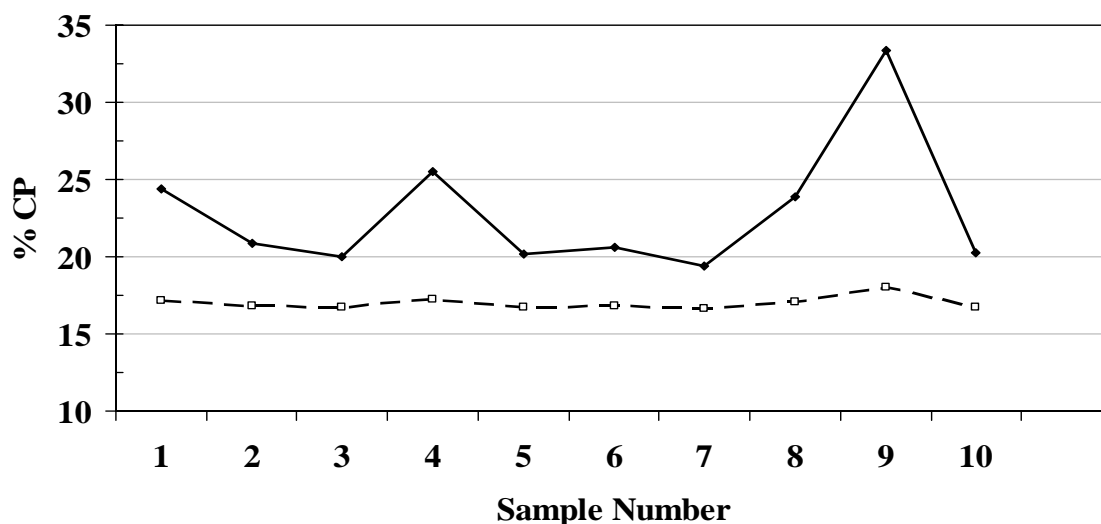
Feed	lbs/day (DM basis)	lbs CP	Variance (x 10,000)
Alfalfa silage	16.8	3.36	282 (39%)
Corn silage	11.2	1.00	25
Ground shelled corn	12.9	1.26	67
Distillers dried grains	6.8	2.06	324 (45%)
Soybean meal	3.6	1.95	25
Minerals-Vitamins	0.9	0	0
Total	52.2	9.63	723

Standard Deviation = 0.27

**Table 6.** Expected mean crude protein level and variance of a simple TMR with forage samples analysed by a laboratory and a multi-component feed made by a feed manufacturer.

Feed	lbs/day (DM basis)	lbs CP	Variance (x 10,000)
Corn silage	16.1	1.45	46.6
Alfalfa silage	8.1	1.61	64.8
Ground shelled corn	6.5	0.63	16.7
Wheat middlings	4.0	0.77	19.2
Ground barley	3.2	0.39	8.0
Distillers dried grains	3.0	0.91	63.0
Corn gluten feed	3.0	0.69	15.3
Alfalfa hay	2.7	0.54	6.0
Soybean meal	2.7	1.42	13.1
Soybean hulls	1.0	0.12	1.0
Canola meal	1.0	0.41	2.5
Corn gluten meal	0.5	0.33	1.0
Total	52.2	9.63	257

Standard Deviation = 0.16



**Figure 5.** Crude protein concentration (dry basis) of 10 loads of corn gluten feed from a single source (solid line; DePeters et al., 2000). The dashed line represents expected concentrations of crude protein in a TMR that was balanced to contain 17 % crude protein using the mean concentration of crude protein in the corn gluten feed (22.9 %) and an inclusion rate of 10 % of the dry matter. Note the small effect that variation in protein concentration of the ingredient has on variation in TMR protein concentration.

### Reducing the Impact of Variation

The composition of all feeds vary. However, the probability that all feeds in a diet will have a lower than expected concentration of a given nutrient on a given day is low. Some feeds will have higher than expected concentrations, others will have lower than expected concentrations. Therefore, the variation in nutrient composition of feedstuffs is usually greater than variation in nutrient composition of the TMR (assuming good, standard feeding practices are in place). The impact of variation in the composition of feedstuffs is reduced as more feeds are included in diets. Relying on a particular feedstuff that is highly variable in CP concentration to provide a large proportion of dietary CP increases the risk of being wrong. We know that, on a theoretical basis the contribution of a feedstuff to the variance of the total diet grows with the square of its inclusion rate (St-Pierre and Harvey, 1986). If that particular feedstuff provided only 10 % of the CP in the diet, a 5 percentage unit change in its CP concentration would cause dietary CP concentration to change by only 0.5 percentage units. In Figure 5, the concentration of CP in different loads of corn gluten feed is shown (DePeters et al., 2000). The CP concentration ranged from 19.4 to 33.4 % (mean = 22.9; SD = 4.3). The load-to-load variation appears quite high. However, if the TMR was balanced for 17 % CP using the mean value for

corn gluten meal and the diet contained 10 % corn gluten (DM basis), the variation in the concentration of CP is much smaller and ranged from 16.6 to 18 % (Figure 5). Using a wide variety of ingredients in a TMR and not relying too heavily on a single ingredient is probably the best way to reduce the costs associated with variation.

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